TUTORIAL: RESOLUTION AND SNR IN FT SPECTROMETRY

The ARCspectro-ANIR and ARCspectro-HT manufactured by ARCoptix are Fourier-Transform (FT) spectrometers. This document briefly describes some basic principles underlying FTS, and focuses on the issues of **resolution** and **signal-to-noise ratio** (SNR).

BASICS

A FT spectrometer is an interferometer. It divides a light beam into two parts and measures the interference that is produced as the two beams are recombined, after one of the beams has been retarded by a variable optical path δ. The measured interferogram $I(\delta)$ and the spectrum $S(\nu)$ of the light source are related through the Fourier cosine transform:

$$S(\nu) = \int_{-\infty}^{+\infty} d\nu\, I(\delta) \cos(2\pi\nu\delta)$$

where $\nu = 1/\lambda$ is the wave-number (or spatial frequency). Below are some schematic examples of spectra and their corresponding interferograms.

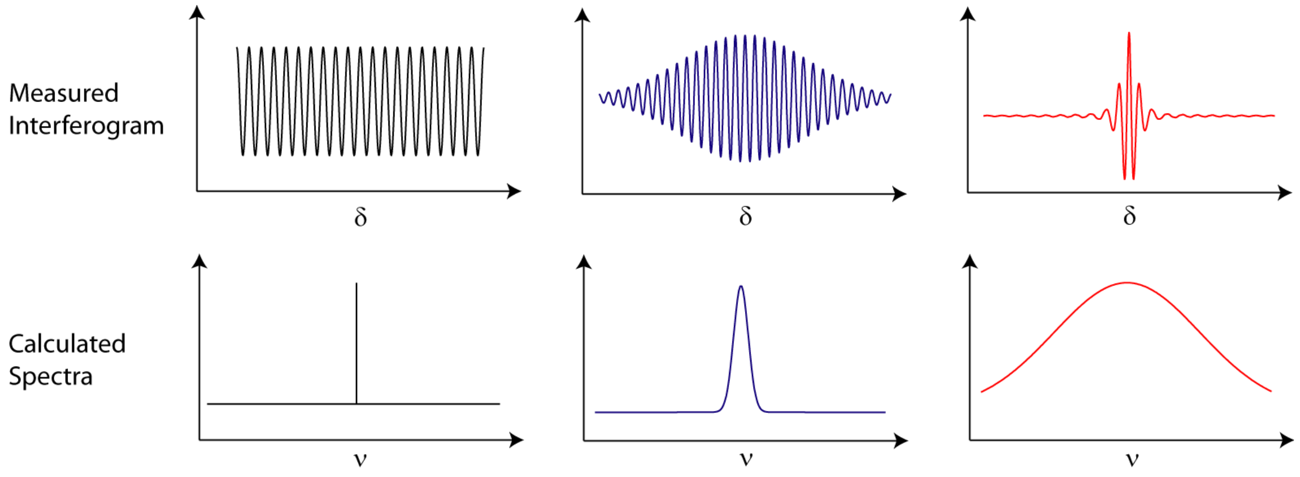

FIGURE 1 - SCHEMATIC SPECTRA AND CORRESPONDING INTERFEROGRAMS

RESOLUTION

The resolution of a FT spectrometer does not depend on a slit width as in the case of a grating spectrometer, but rather depends on the maximal retardation, or Optical Path Difference (OPD), achievable by the scanning mechanism of the interferometer ($OPD = 0.5mm$ in the ARCspectro ANIR series). The inevitably limited scanning range implies that only a finite part of the interferogram can be recorded. Thus the integral bounds in the Fourier transform have to be changed to:

$$S(\nu) = \int_{-OP\mathrm{D}}^{+OP\mathrm{D}} d\delta\, I(\delta) \cos(2\pi\nu\delta)$$

When considering a monochromatic light source centered at ν_0 (that would produce a simple cosine interferogram with a period equal to $\lambda_0 = 1/\nu_0$) the calculated spectrum $S(\nu)$ is no longer an infinitesimally sharp peak. It is a *sinc* function, shown on Figure 2. It defines the finite resolution achievable by the instrument.

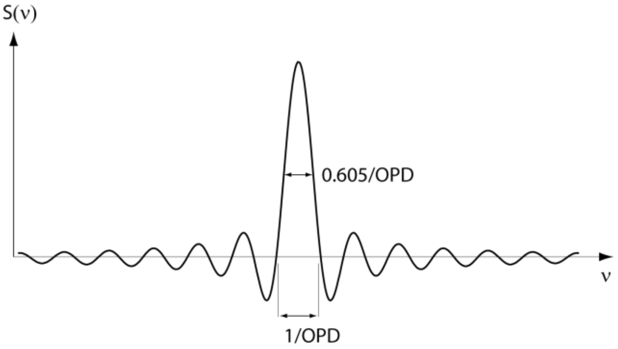

FIGURE 2 - RESPONSE TO A MONOCHROMATIC LIGHT SOURCE WITH A FINITE OPTICAL PATH DIFFERENCE (OPD)

The theoretical full-width half-maximum (FWHM) resolution of a FT spectrometer (in wave-numbers υ) is thus given by:

$$\Delta\nu \approx \frac{0.605}{OPD}$$

When translated into **wavelengths**, the following formula applies for the theoretical resolution:

$$\boxed{\Delta\lambda \approx 0.605\frac{\lambda^2}{\mathrm{OPD}}}$$

Note that **the resolution of a FT spectrometer is wavelength-dependent:** it is finer at the shorter wavelengths than at the longer wavelengths.

For the ANIR 0.9-2.6, which is capable of imposing a maximal OPD of $0.5\mathrm{mm}$ the following wavelength resolutions are possible at different wavelengths:

Wavelength λ	1000nm	1700nm	2600nm
Resolution Δλ	1.21nm	3.49nm	8.17nm

Practically, the spectrum calculated from the interferogram using a fast Fourier transform (FFT) algorithm, that produces data points at discrete, equally spaced *wave-numbers* υ_k. However, when displaying spectral information on a *wavelength* scale, data points are located at the corresponding wavelengths $\lambda_k = 1/\nu_k$, which are *not* equally spaced. This is illustrated in Figure 3.

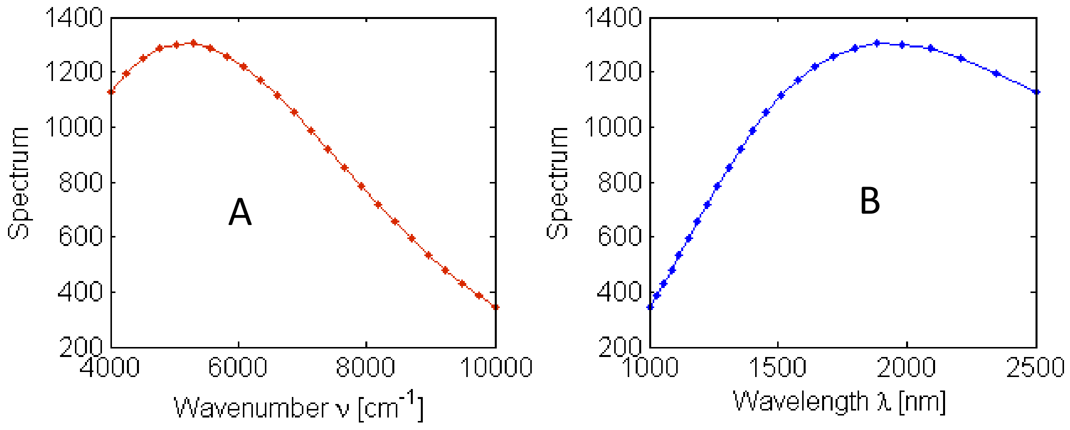

FIGURE 3 - DATA POINT SPACING (A) IN WAVE-NUMBER DOMAIN AND (B) IN WAVELENGTH DOMAIN

When plotting data in the wavelength domain (Fig. 3b), data points are closely spaced at the shorter wavelengths, and progressively more and more distant towards the longer wavelengths. Still, the resolution achieved by the ARCspectro ANIR 0.9-2.6 at the longer wavelengths is comparable to the resolution of a diffraction grating spectrometer equipped with a InGaAs 256-element detector array, while it is much finer for the ANIR at the shorter wavelengths.

SIGNAL-TO-NOISE RATIO

In a FT spectrometer the *interference* between all wavelengths is measured as a function of the retardation δ, meaning that *all wavelengths are always simultaneously impinging onto the detector*. This has consequences on the signal-to-noise ratio (SNR), as the photo-detector can only accept a fixed amount of optical power before being saturated.

Figure 4 schematically compares a *broad*-bandwidth light source (red line) to a *narrow*-bandwidth light source (blue line). The optical power impinging on the detector, which equals to the area underlying the graph, is the same in both scenarios. However, the narrower is the bandwidth, the higher is the spectral irradiance (irradiance per nm) that can be accepted by the detector before it is saturated. In other words, a higher optical signal can be accepted for a restricted bandwidth. As the detector noise signal stays constant, **the achievable SNR is higher when measuring a source with a *narrow* bandwidth than when measuring a source with a *broad* bandwidth.**

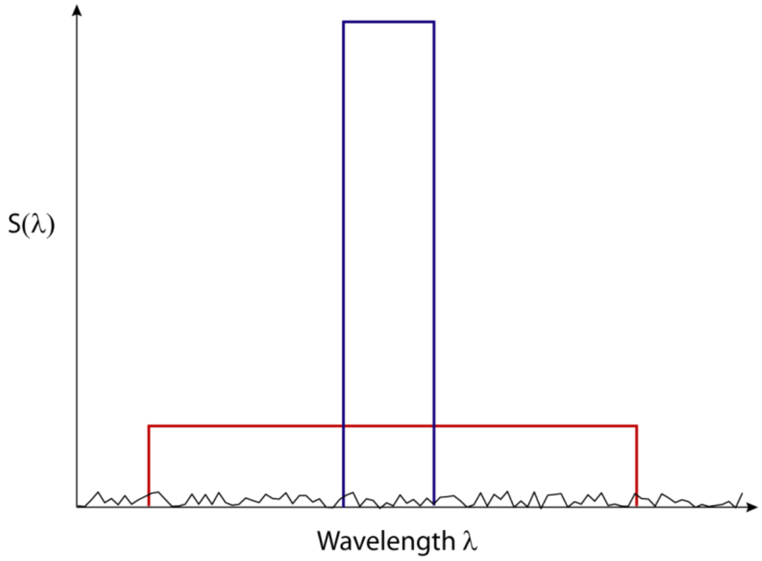

FIGURE 4 - MEASUREMENT BANDWIDTH AND SNR

Practically, when measuring a broadband light source, such as a halogen light bulb, a SNR in the order of 1000 can be achieved with the ARCspectro ANIR (ratio between the strongest signal in the spectrum to the noise level). The SNR can sometimes be seen to degrade at the shorter wavelengths (below 1200nm). This does not mean that the instrument is not capable of measuring spectral information at the shorter wavelengths: the signal is simply “drowned” in the large amount of light emitted at the longer wavelengths by the halogen light bulb. **High-SNR measurements can still be performed below 1200nm by inserting a low-pass filter in the light path**, thereby limiting the measurement bandwidth. In a general manner, higher SNR in any wavelength range can always be achieved by means of band-pass filters, allowing the instrument to “concentrate” on the user defined wavelength range.

Also, when measuring sources with a naturally narrow bandwidth, such as lasers or gas discharge lamps, the ARCspectro ANIR is capable of achieving huge SNR in the order of 10’000 to 100’000.